Assignment 3

Problem 3.4 provides eight extra credit points. Thus, there are 32 points achievable on this assignment, but only 24 points are accounted for as 100%.

3.1. Problem (8)

Butterfly networks

Prove the following statements:

(a) The butterfly network $B_d$ and the wrapped butterfly have the same number $d \cdot 2^{d+1}$ of edges and both have degree 4. $B_d$ has $(d + 1) \cdot 2^d$ nodes, the wrapped butterfly has $d \cdot 2^d$ nodes.

(b) The diameter of $B_d$ is $2 \cdot d$ and decreases to $\lfloor \frac{3d}{2} \rfloor$ for the wrapped butterfly.

3.2. Problem (8)

Randomized routing on the hypercube

Consider the randomized routing algorithm on the hypercube (Algorithm 2.19 on p. 50). Show that there is a packet $P_w \in \mathcal{P}$, that blocks $P_v$ at most once.

3.3. Problem (8)

The Message Passing Interface

Show how to construct paths $v \rightarrow v \oplus b$ in the $d$-dimensional hypercube for each $b \in \{0, 1\}^d$, such that no edge is used by more than one path in the same direction.

(As a consequence several collective communication functions can be implemented on the hypercube without any congestion.)

3.4. Problem (8)

Independent Set

Construct an independent set of maximum size for $d$-dimensional hypercube $Q_d$. An independent set is a subset $I \subseteq \{0, 1\}^d$ of vertices such that no two vertices in $I$ are connected by an edge.