Assignments 10

Note: It is understood that all of your statements have to be proven correct.

Note: Solutions may be submitted by email. Solutions submitted after the lecture will not be graded.

Exercise 10.1. (8)

Random walks in directed graphs

Prove that the cover time for a directed graph $G$ can be exponential in the size of $G$.

Exercise 10.2. (8)

Universal traversals

Consider the class $\mathcal{G}(n,m)$ of undirected connected graphs on $n$ nodes and $m$ edges. A universal traversal is a sequence $t = t_1 \ldots t_k \in \{1, \ldots, n\}^k$, where $k = O(poly(n,m))$, such that for any $G \in \mathcal{G}(n,m)$ it holds that $t$ visits all nodes when starting at an arbitrary node of $G$. The traversal rule for the current node $v$ at step $i \geq 1$ is: hop to $v$’s neighbor number $t_i \mod \text{deg}(v)$.

Prove that universal traversals exist.

Hint: Show that $\text{prob}(t \text{ is universal}) > 0$ if $t$ is picked uniformly at random from $\{1, \ldots, n\}^k$, where you determine $k$. Think about concatenating random walks. Which length should each of them have? How many are needed?

Exercise 10.3. (8)

$k$-SAT

We analyze a version of Algorithm 3.17 for which the loop in step (3) is iterated $6n$ times. In this setting the result of Lemma 3.18 can be shown analogously by means of the following statement:

Let a random walk start in state $d$. If the probability to walk left (i.e., to decrease the state number) is at least $\frac{2}{3}$, then the walk reaches state 0 within $6d$ steps with probability at least $\frac{1}{2}$.

Prove the statement.